Design of realisable optimum post-detection filters for high data-rate *m*-PAM systems with clock jitter tolerance

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The continual increase in demand for extremely high data-rate systems spans the full range of network systems from local area networks to long-haul systems. While coherent optical systems dominate the long-haul market, there is significant interest in using simpler direct detection systems for application areas, where simplicity and lower-cost is a key driver. Pulse-amplitude modulation (PAM) is appropriate to consider as it enables the reduction of the channel symbol rate for a given data rate, without unduly increasing the complexity of the system. A route to the design of optimum post-detection filter for PAM systems is presented that offers significantly improved robustness to timing jitter, which is an impairment of increasing import as symbol rates are increased.

Introduction: The push to ever higher data rates in communications systems, driven by the accelerating growth in data traffic, is clearly going to continue for the foreseeable future. This growth will be seen across all parts of the network encompassing LANs, metropolitan area networks (MANs), and long-haul systems. Long-haul links are dominated by coherent optical systems that can utilise complex modulation schemes such as m-QAM. The retention of both amplitude and phase information post-detection in coherent systems enables the use of very complex signal processing to enable the correction of the majority of fibre impairments such as dispersion and non-linear behaviour.

Simpler, envelope-based modulation schemes such as pulse-amplitude modulation (PAM) are of particular interest for shorter reach systems such as MANs and LANs, where the choice of modulation scheme is driven by the need to reduce the symbol rate while managing overall system complexity so that a cost-effective solution can be delivered. PAM is, for example, being considered as one of the possible technologies for terrabit Ethernet (TbE) [1].

As the symbol rate increases, jitter in the recovered clock inevitably becomes more of an issue, and in order to achieve optimum performance in the presence of clock jitter the design of the system has to ameliorate as far as possible the impact of clock jitter. The challenge for the system designer here involves trading off the impact of the post-detection filtering on the noise presented to the decision circuit, the intersymbol interference (ISI), and the impact of timing errors in the sampling of the received signal caused by clock jitter. Adoption of a filtering strategy to yield a 100% raised-cosine spectrum at the input to the decision device yields a zero ISI signal, but the signal is very fragile in the presence of timing errors and is not optimum in terms of noise filtering.

Earlier work [2] used a range of techniques to design optimum filters for intensity modulated direct detection systems using on–off keying, and here the approach is extended to demonstrate the design of realisable filters for m-PAM systems that achieve optimum performance taking into consideration noise, ISI, and clock jitter.

Approach: An m-PAM system simulator was developed in Gnu Octave [3]. In the simulator, a random binary bit sequence was grouped into blocks of $\log_2 m$ bits and then mapped onto one of the m levels. Each of these symbols was then repeated n times to yield a sampled version of an m-PAM sequence with n samples per symbol. The white Gaussian noise was added to the samples to achieve a predefined signal-to-noise ratio (SNR). The noisy signal was then filtered and subsequently sampled at the point of maximum signal amplitude for each symbol. The samples were converted back into blocks of $\log_2 m$ bits by comparison against a set of thresholds and the resultant bit stream compared with the original random sequence in order to determine the BER.

To ensure that the filters designed were realisable, they were defined as simple all-pole filters. For the work presented here, three pole filters were adopted where the pole locations were at $s = -\sigma_1$ and $s = -\sigma_2 \pm j\omega$. The three parameters $\{\sigma_1, \sigma_2, \omega\}$ then fully define the filter. Filters of this form are easily realisable as planar microwave structures [4], or the response can, by appropriate design, be realised directly in the amplifier at the receiver [5].

The impact of clock jitter was modelled by perturbing the sampling time away from the optimum sampling time with a random offset t_j uniformly chosen from the range $(-t_{j,\max}, \ t_{j,\max})$. $t_{j,\max}$ was normalised to the symbol period.

To identify the optimum filter parameters, an optimisation algorithm was used to find $\min\{\mathrm{BER}(\sigma_1, \sigma_2, \omega) \mid \mathrm{SNR}, t_{j,\max}\}$ for a range of SNR and maximum jitter values with a constraint that σ_1, σ_2, ω are all >0 to ensure that the post-detection filter is realisable.

The error space being explored by the optimisation algorithm is very complex, with many local minima. A simulated annealing [6] algorithm was used with the parameters tuned to drive convergence in a reasonable time while still adequately exploring the problem space. Even though run times were long – with each filter design taking about 5 days on a high-end desktop personal computer – the use of annealing enabled the optimisation to proceed unsupervised, whereas a downhill optimiser would need to be started from many places manually to try to understand the problem space and to identify a good final solution. Octave is single threaded, but many instances can be run simultaneously on a multi-core machine so that several filters can be designed in parallel.

Illustrative results: To illustrate the benefits that the adoption of this approach can yield a set of optimum filters were designed for a 16-PAM system with an SNR of 30 dB with normalised maximum jitter values in the range 0–0.3.

As an example, for this 16-PAM system the optimum filters shown in Table 1 were identified for the indicated normalised maximum jitter values.

Table 1: Optimised pole locations for 16-PAM system with range of normalised maximum jitter values and SNR = 30 dB

Normalised Jitter	σ_1	σ_2	ω
0	7.52	5.42	3.72
0.1	13.68	7.66	3.95
0.2	15.34	12.49	4.05
0.3	19.93	18.45	1.16

Of key import is the robustness of the filter designs to variations in clock jitter. To quantify this tolerance, the Octave code was used again to evaluate the performance of each of the four filter designs that were designed for specific maximum jitter values across the full range of maximum jitter values. Fig. 1 shows the resulting BER against jitter for the four filters itemised in Table 1.

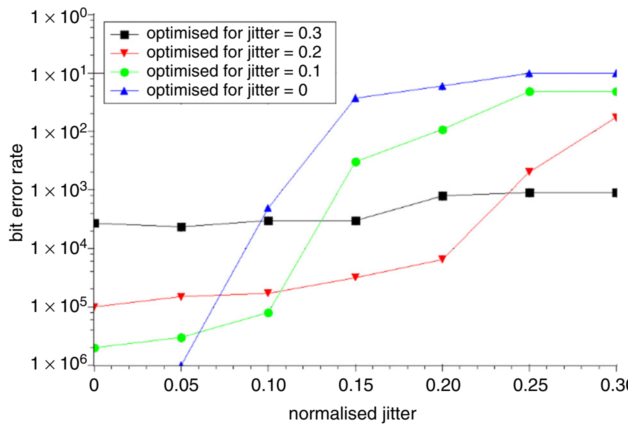

Fig. 1 *BER for four filter designs against normalised jitter*

Each filter designed for a specific maximum jitter value is the best performer at that particular jitter. Also, the filters designed for lower jitter values clearly outperform those designed for high-jitter values when jitter levels are lower than the design target. However, it is clear that the filters designed for low-jitter values do not display robustness to increasing jitter and rapidly yield BERs worst than the widely adopted target of 1×10^{-3} at higher jitter levels. On the other hand, the filters designed for normalised jitter values of 0.2 and 0.3, even though they are outperformed by the low-jitter filters at low-jitter levels, deliver a usable performance with a BER better than 1×10^{-3} to jitter levels out to 0.3.

It is informative to look at the eye diagrams for the two extreme cases (the filter optimised for $t_{j,\max} = 0$ and for $t_{j,\max} = 0.3$.

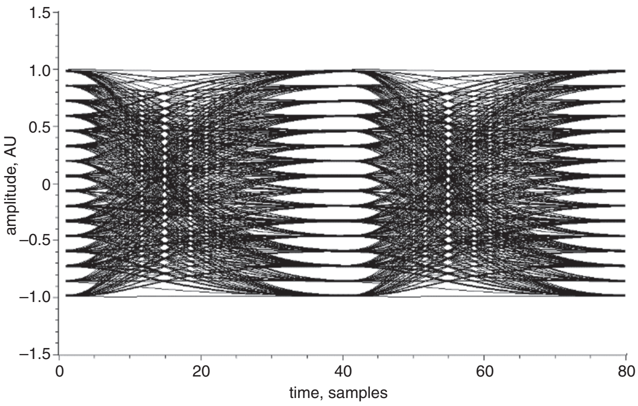

Fig. 2 *Eye diagram for 16-PAM signal at SNR = 30 dB, filter optimised for zero jitter*

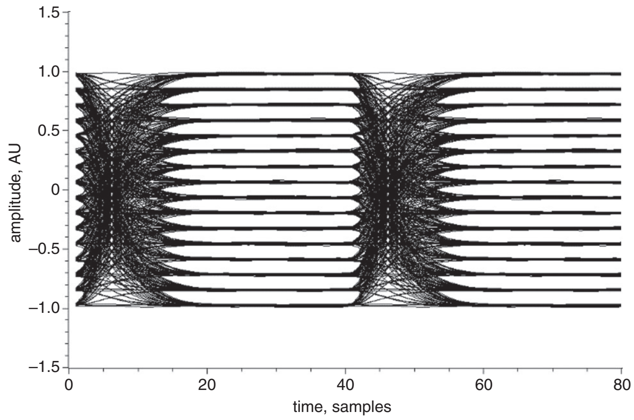

Fig. 3 *Eye diagram for 16-PAM signal at SNR = 30 dB, filter optimised for jitter = 0.3*

Comparing Figs. 2 and 3, it can be seen that the signal at the output of the filter that is optimised for zero jitter has a better maximum vertical eye opening when compared with the filter optimised for a jitter of 0.3. This explains the better BER performance of this filter at zero or low-jitter levels, where the signal is sampled at, or close to, the optimum point. On the other hand, the filter that is optimised for high-jitter levels has a much wider horizontal eye opening which explains the vastly improved robustness to large jitter levels. This behaviour can be explained by considering the pole locations for the filters in Table 1. The high-jitter filter has its poles further from the origin, which opens up the bandwidth of the filter. This wider bandwidth filters less noise, hence the poorer performance in the absence of jitter, but it improves horizontal eye opening as the spectrum of the received pulses is less constrained. The frequency response of the four filters is compared in Fig. 4. The optimisation leads to actual pole locations that ensure that the balance between the three factors that influence performance – noise filtering, horizontal eye opening and ISI – is such that optimum overall performance is delivered.

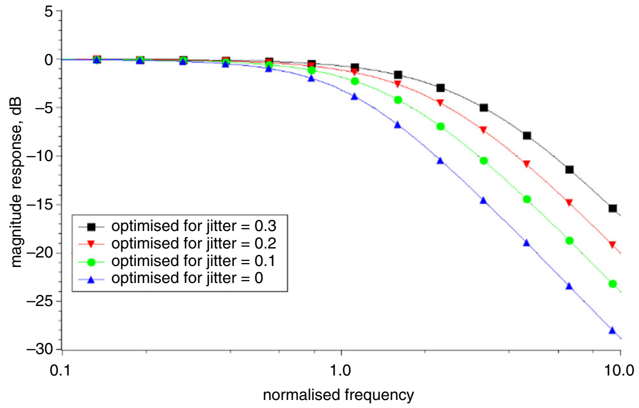

Fig. 4 *Magnitude frequency response of four optimised filters*

It should be emphasised again that the filters designed by the adoption of this approach are readily realisable, and therefore can be used in deployed systems.

Conclusion: This Letter has presented an approach to the design of realisable, optimum post-detection filters for *m*-PAM systems and illustrated the performance improvement that such filters can deliver in a high data-rate *m*-PAM system. The useful robustness to jitter achieved should support the pushing of *m*-PAM systems to higher data rates to meet emerging needs in usage scenarios such as TbE.

Submitted: *14 February 2018*
doi: 10.1049/el.2018.0238
One or more of the Figures in this Letter are available in colour online.

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